

Color confinement and dual superconductivity in unquenched QCD

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We report on evidence from lattice simulations that confinement is produced by dual superconductivity of the vacuum in full QCD as in quenched QCD. Preliminary information is obtained on the order of the deconfining phase transition.

1. Introduction

Confinement of color and the deconfining phase transition have been observed in numerical simulations on the lattice. In pure gauge theory, (SU(2) and SU(3)), the transition is well understood. $\langle L \rangle$, the expectation value of the Polyakov line, is a good order parameter and $\langle \tilde{L} \rangle$, the expectation value of the 't Hooft line, is a good disorder parameter [1]; the corresponding symmetries are Z_N , \tilde{Z}_N .

The situation is less clear in the presence of dynamical quarks (full QCD). There the symmetry Z_N (\tilde{Z}_N) is explicitly broken by the coupling of the quarks. In the chiral limit $m_q = 0$ the chiral parameter $\langle \bar{\psi}\psi \rangle$ would be a good order parameter for the chiral phase transition; however at $m_q \neq 0$ also chiral symmetry is explicitly broken. Moreover it is not clear a priori what relation exists between chiral symmetry and confinement.

For a model QCD with two quarks ($N_f = 2$) of equal mass $m_u = m_d = m_q$ the phase diagram is schematically represented in Fig. 1. The line of phase transition in the plane m_q, T is defined by the maxima of susceptibilities [2,3], among which

$$\chi_L = \int d^3x \langle L(\vec{x}, 0) L^\dagger(\vec{0}, 0) \rangle \quad (1)$$

and the susceptibility χ_{ch} of the chiral order parameter

$$\chi_{\text{ch}} = \int d^3x \langle \bar{\psi}\psi(\vec{x}, 0) \bar{\psi}\psi(\vec{0}, 0) \rangle . \quad (2)$$

All of them have a maximum at the same value of T , for a given m_q , which defines the line in Fig. 1. For $m_q > 3$ GeV, the maxima of χ_L diverge proportionally to the volume V ,

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indicating a first order transition. At $m_q = 0$ there are theoretical reasons and numerical indications that the transition is second order [4,2,3]. At intermediate values of m_q the susceptibilities studied in [2,3] stay finite as $V \rightarrow \infty$, and this is interpreted as a crossover.

In the usual approach to phase transitions the free energy (effective lagrangean in the language of field theory) depends on the order parameters in a form which is dictated by symmetry and by scale analysis. The susceptibilities which keep memory of the order of the transition are those of the order parameters. At intermediate quark masses neither $\langle L \rangle$ nor $\langle \bar{\psi}\psi \rangle$ are good order parameters.

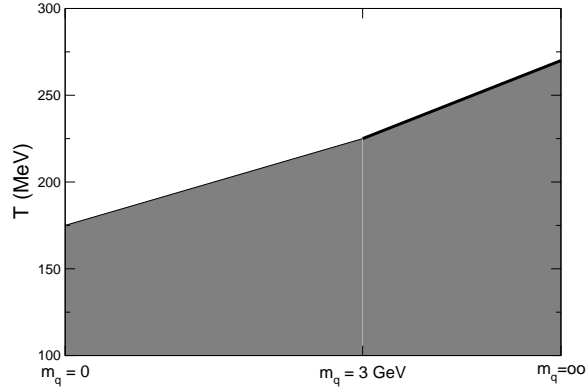


Figure 1. Phase diagram for two degenerate flavours, $m_u = m_d = m_q$.

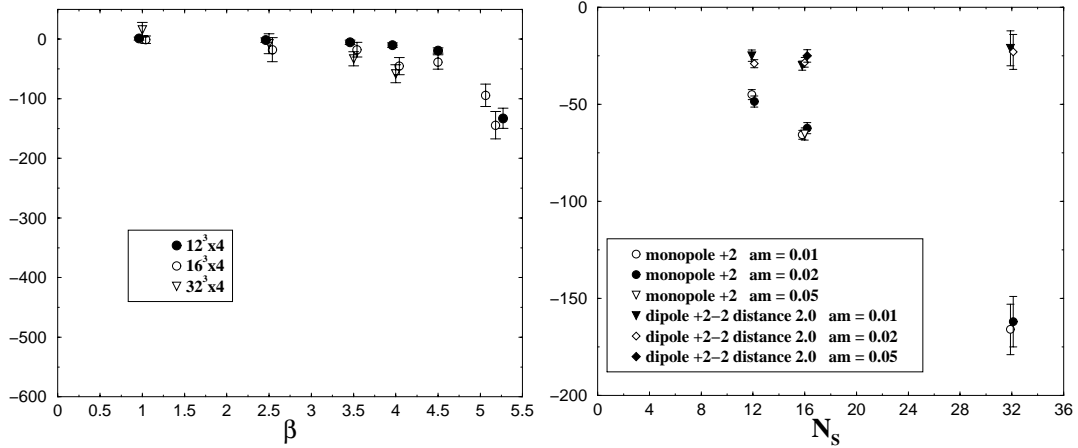


Figure 2. ρ as a function of N_s for $T < T_c$ (left-hand side). ρ as a function of N_s for $T > T_c$: as $N_s \rightarrow \infty$, $\rho \rightarrow -\infty$ if the magnetic charge is non-zero and stays constant otherwise (right-hand side).

A good order parameter exists, which is the vacuum expectation value of a magnetically charged operator μ . $\langle \mu \rangle \neq 0$ signals dual superconductivity of the vacuum [5–8]. According to the ideas of ref.'s [9,10], dual superconductivity channels the chromoelectric field acting between a $q\bar{q}$ pair into an Abrikosov flux tube, with energy proportional to the length, $V = \sigma r$.

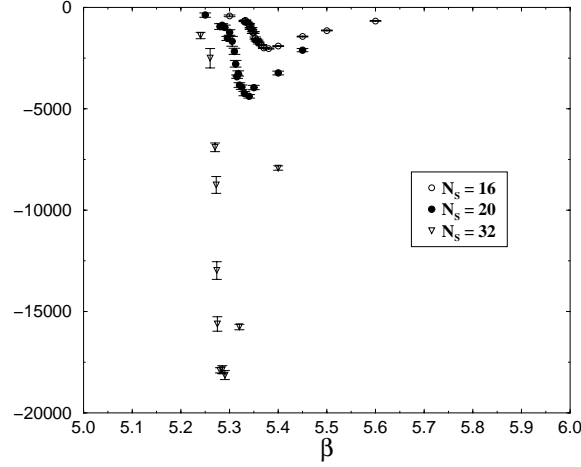


Figure 3. ρ peak at different values of N_s . The values of m_q are chosen such that $m_q N_s^\gamma$ is kept constant ($m_q = 0.075$ at $N_s = 16$, $m_q = 0.043$ at $N_s = 20$ and $m_q = 0.01335$ at $N_s = 32$).

In quenched QCD [6–8] a detailed analysis of $\langle\mu\rangle$ shows that the vacuum is indeed a dual superconductor for $T < T_c$ ($\langle\mu\rangle \neq 0$), and makes a transition to normal at T_c . Above T_c , $\langle\mu\rangle = 0$. As $T \rightarrow T_{c-}$, $\langle\mu\rangle \simeq \tau^\delta \phi(a/\xi, \xi/N_s)$, with $\tau = (1 - T/T_c)$, $\xi \sim \tau^{-\nu}$ the physical correlation length and N_s the linear spatial extension of the lattice. At the critical point $a/\xi \ll 1$ and can be put to zero, while the variable ξ/N_s can be traded with $N_s^{1/\nu} \tau$. As a consequence the scaling law $\langle\mu\rangle = \tau^\delta \tilde{\phi}(N_s^{1/\nu} \tau)$ holds. A better variable than $\langle\mu\rangle$ is $\rho = \frac{\partial}{\partial \beta} \log \langle\mu\rangle$, which is a susceptibility. In terms of ρ ,

$$\langle\mu\rangle = \exp \left(\int_0^\beta \rho(\beta') d\beta' \right) \quad (3)$$

The scaling law $\rho/N_s^{1/\nu} = f(\tau N_s^{1/\nu})$ allows one to control the infinite volume limit $N_s \rightarrow \infty$, which is necessary to have a phase transition, and to extract ν , T_c , δ . For quenched QCD, $\langle\mu\rangle$ is equally good as $\langle\tilde{L}\rangle$ and is numerically coincident with it [1].

A similar analysis can be repeated for full QCD [11]. $\langle\mu\rangle$ is defined as in quenched and defines the same symmetry. Simulations have been done on $N_t \times N_s^3$ lattices with $N_t = 4$ and $N_s = 12, 16, 20, 32$. The time extension N_t determines the transition temperature, the space extension N_s is used to perform the finite size scaling analysis of the infinite volume limit. The first result is that for $T < T_c$, $\langle\mu\rangle \neq 0$ as $N_s \rightarrow \infty$. Indeed in that range of temperature, ρ is N_s independent (see Fig. 2). For $T > T_c$ instead $\rho \simeq -kN_s + c$ ($k > 0$) as $N_s \rightarrow \infty$ (see Fig. 2), i.e., by Eq. (3), $\langle\mu\rangle$ is strictly zero in the infinite volume limit.

Around T_c , again,

$$\langle\mu\rangle = \tau^\delta \phi \left(\frac{a}{\xi}, \frac{N_s}{\xi}, m_q N_s^\gamma \right). \quad (4)$$

A new scale, m_q , appears which was absent in the quenched case. However the exponent γ is known ($\gamma \simeq 2.49$ [2,3]) and simulations can be made choosing m_q and N_s such that

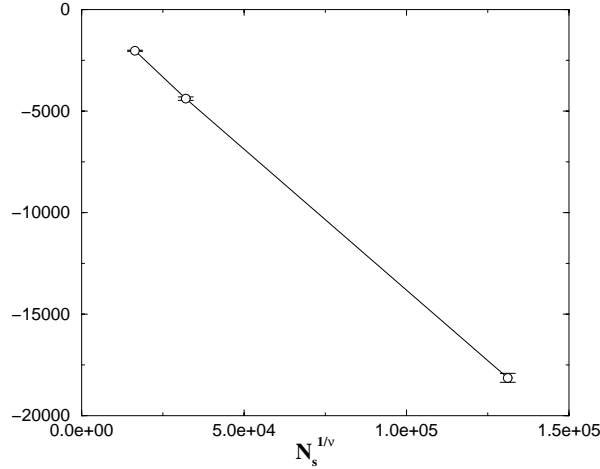


Figure 4. Values of ρ at the peak as a function of $N_s^{1/\nu}$, with $\nu = 1/3$. The height of the peak clearly scales as $N_s^{1/\nu}$.

$m_q N_s^\gamma$ in Eq. (4) is kept fixed. Then, neglecting a/ξ again, the scaling law becomes

$$\langle \mu \rangle = \tau^\delta \phi \left(0, \frac{N_s}{\xi}, \text{const.} \right), \quad \rho / N_s^{1/\nu} = f(\tau N_s^{1/\nu}). \quad (5)$$

The behaviour of ρ around T_c is shown in Fig. 3 for different values of N_s : it has a peak at T_c , Eq. (5) implies in particular that the values of ρ at the peak ($\tau = 0$) $\bar{\rho}$, scale as $\bar{\rho} \propto N_s^{1/\nu}$, whence ν can be determined. The result is shown in Fig. 4: contrary to the common belief, the transition is first order. This result is still preliminary and will be cross-checked by more simulations, and by the use of improved actions. A scenario in which confinement is controlled by one single order parameter $\langle \mu \rangle$ and by one and the same symmetry pattern (dual superconductivity) is appealing and meets the requirements of $N_c \rightarrow \infty$ arguments.

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